EXCITATION OF OSCILLATIONS IN A GAS-BLOWER – FLUIDIZED-BED SYSTEM

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It is shown that hydraulic characteristics in a fluidized bed oscillate in a spectrum that is discrete due to the reciprocal effect of the fluidization regime on gas-blower operation.

Fluidization at any fluidizing-agent filtration velocity is accompanied by more or less intensive, stable pulsations of the hydrodynamic parameters. Here, aggregates of particles (packets) and low-density zones (bubbles) periodically form and disintegrate in the bed. The motion of the particles is random and is characterized by small-scale pulsations. Superimposed on these pulsations are low-frequency oscillations of global characteristics of the bed such as its total height and the pressure drop in the bed. Present discussion revolves around the existence of a set of mechanisms responsible for these pulsations, and all of them evidently actually exist in fluidized beds [1-3]. Foremost among these mechanisms is turbulent pulsation of the fluidizing gas, which generates small-scale pulsations of particle motion. A second mechanism consists of pulsations caused by nonuniformity of the speed of the rotor. These two types of pulsations are characterized by a relatively high frequency and low energy. From a practical standpoint of fluidized bed use, it is most important to study low-frequency pulsations occurring in the bed due to the presence of collective effects (packets, bubblers) in it. According to current representations, these collective effects are generated by the system itself (the model of a "gravitational pendulum" with a frequency $f \sim \sqrt{g/h}$ [1]) or are a consequence of the effect of the volume of the space under the grate – which acts as a resonator – on the fluidization regime [2, 4, 5]. A fluidized bed, being a medium with distributed inertia and elasticity, also permits the propagation of pressure, concentration, and velocity waves in it (these waves being nonlinear in the general case).

The present study examines one more mechanism of oscillation generation in a fluidized bed. This mechanism is associated with the reciprocal effect of the fluidization regime on the operation of the gas blower. Henceforth in studying the stability of homogeneous, steady-state fluidization (which is actually not realized) in which a uniform flow of fluidizing gas keeps stationary particles in a suspended state at certain distances from one another, we write the unidimensional equations of motion and continuity for the particles in a continuum approximation:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = -g + F(v,\ \varepsilon,\ u),$$
$$\frac{\partial \varepsilon}{\partial t} + v\frac{\partial \varepsilon}{\partial z} = (1-\varepsilon)\frac{\partial v}{\partial z}. \tag{1}$$

The fluidizing gas will be assumed to be incompressible and moving at a constant velocity u over the cross section of the unit, the velocity being calculated for an empty unit. Also assuming the particles to be incompressible and ignoring mass exchange between the phases and particle fragmentation and coalescence, we obtain the following condition for conservation of the total volume of particles in a bed of height h

$$\int_0^h (1-\varepsilon)\,dz = h_0(1-\varepsilon_0) = \mathrm{const}. \tag{2}$$

Ignoring the compressibility of the gas, we write the equation expressing the feedback in the gas-blower–fluidized-bed system in the form

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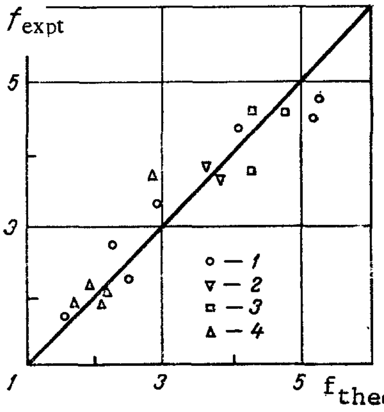

Fig. 1. Comparison of experimental and theoretically calculated frequencies of oscillation in different fluidized beds. The material of the particles – glass: 1) d = 0.9; 4) 0.5 mm; corundum: 2) d = 0.5; 3) 0.63 mm, f_{expt}; f_{theor}, Hz.

$$\Delta p_e = \Delta p_T + \int_0^h F(v, \varepsilon, u)\,dz. \tag{3}$$

Here, Δp_e is the pressure drop created by the source of the gas flow. Its magnitude is usually prescribed by the head characteristic, which is described by a quadratic flow-rate function [6]. The resistances along the bed and the local resistances Δp_t are also generally quadratic functions of the flow rate of the gas. The integral in (3) gives the resistance to the flow of the fluidizing gas attributable to a fluidized bed of height h. There is a sufficient number of experimental approximations and theoretical relations for the function F(v, ε, u), which describes the local hydrodynamic interaction between the fluidizing gas and the particles. We will use Ergan's empirical formula [7], it being the most accurate and being applicable to a broad range of fluidization regimes. However, we will assume that it is applicable not only to the stationary bed as a whole, but also locally, i.e., if the investigated section of a bed of thickness Δz is moving at a velocity v and the gas inside this bed is moving at the velocity u/ε, then the resistance of the bed will be:

$$F(v, \varepsilon, u) = 150\,\frac{\mu_0(1-\varepsilon)^2}{d^2\varepsilon^2}\left(\frac{u}{\varepsilon} - v\right) + 1{,}75\,\frac{\rho_0(1-\varepsilon)}{d\varepsilon}\left(\frac{u}{\varepsilon} - v\right)^2. \tag{4}$$

Equations (1-4) constitute a system of unidimensional equations of a fluidized bed with allowance for the reciprocal effect of the fluidization regime on the operation of the gas blower. Henceforth, for convenience we will change over to dimensionless variables by means of the relations

$$\eta = \varphi^2 gz,\quad \tau = \varphi gt,\quad U = \varphi u,\quad V = \varphi v,\quad H = \varphi^2 gh,$$
$$B = \frac{b}{\rho_1\varphi g},\quad K_1 = \frac{k_1\varphi^2}{\rho_1},\quad K_2 = \frac{k_2\varphi}{\rho_1},\quad K = \frac{k_3 + k_4}{\rho_1}, \tag{5}$$

where

$$\varphi = \frac{1{,}75}{150}\,\frac{\rho_0 d}{\mu_0};\quad b = 150\,\frac{\mu_0}{d^2}.$$

Then system (1-4) takes the form

$$\frac{\partial V}{\partial \tau} + V\frac{\partial V}{\partial \eta} = -1 + \frac{B}{\varepsilon}\left(\frac{1-\varepsilon+U}{\varepsilon} - V\right)\left(\frac{U}{\varepsilon} - V\right),$$
$$\frac{\partial \varepsilon}{\partial \tau} + V\frac{\partial \varepsilon}{\partial \eta} = (1-\varepsilon)\frac{\partial V}{\partial \eta}, \tag{6}$$
$$\int_0^H (1-\varepsilon)\,d\eta = H_0(1-\varepsilon_0),\quad K_1 + K_2U - KU^2 = H_0(1-\varepsilon_0) + \int_0^H (1-\varepsilon)\left(\frac{\partial V}{\partial \tau} + V\frac{\partial V}{\partial \eta}\right)d\eta.$$

Without consideration of the feedback (the fourth equation of system (6)), the above-formulated problem is analogous to the problem of forced oscillations of a bed that was examined in [3]. In that study, however, phase interaction was described by a different method than here. Allowing for feedback significantly alters the solution of the system. In particular, as will be shown below, spontaneous oscillations of the gasdynamic quantities with a discrete set of frequencies and wave numbers develop in the system.

We will study the behavior of the solution of system (6) near a homogeneous steady state which, as is easily seen, is determined by the relations

$$V = V^0 = 0, \quad U = U^0, \quad \varepsilon = \varepsilon^0, \quad H = H^0, \tag{7}$$

where ε^0 and U^0 are connected by the relations

$$(\varepsilon^0)^3 + BU^0\varepsilon^0 - BU^0(1 + U^0) = 0, \tag{8}$$

which here is nothing more than the well-known relation between fluidization velocity and bed expansion.

The solution of (7) for system of nonlinear equations (6) is a singular point. The type of singularity and its stability can be determined after linearization of the initial system of equations in the neighborhood of the singular point [8]. Introducing small deviations from the steady-state solution V', U', ε', and H', we obtain the following from (6)

$$\frac{\partial V'}{\partial \tau} = pU' - q\varepsilon' - p\varepsilon^0 V', \quad \frac{\partial \varepsilon'}{\partial \tau} = (1 - \varepsilon^0)\frac{\partial V'}{\partial \eta},$$
$$\int_0^{H^0} \varepsilon' d\eta = (1 - \varepsilon^0) H', \quad U' = r \int_0^{H^0} \frac{\partial V'}{d\tau} d\eta, \tag{9}$$

$$p = \frac{1 - \varepsilon^0 + 2U^0}{U^0(1 - \varepsilon^0 + U^0)}; \quad q = \frac{3 - 2\varepsilon^0 + 3U^0}{\varepsilon^0(1 - \varepsilon^0 + U^0)}; \quad r = \frac{1 - \varepsilon^0}{K_2 - 2KU^0}. \tag{10}$$

System (9) is satisfied by solutions in the form of waves for a particle velocity $V'(\eta, \tau)$ and for porosity $\varepsilon'(\eta, \tau)$:

$$V'(\eta, \tau) = \frac{pU_0'}{q(1 - \varepsilon^0)} \frac{\Omega}{\psi} [1 - \exp(-i\psi\eta)] \exp(i\Omega\tau),$$
$$\varepsilon'(\eta, \tau) = \frac{p}{q} U_0' \exp[i(\Omega\tau - \psi\eta)] \tag{11}$$

with the boundary condition $V'(\eta = 0) = 0$. The quantity U_0' is the amplitude of the perturbation of fluidized-bed velocity. The wave vector ψ is related to the frequency Ω by the equation

$$\psi = \Omega \frac{p\varepsilon^0 + i\Omega}{q(1 - \varepsilon^0)}, \tag{12}$$

from which we find expressions for the complex phase and group sonic velocities in the fluidized bed:

$$a_{\mathrm{ph}} = \frac{\Omega}{\psi} = \frac{q(1 - \varepsilon^0)}{p\varepsilon^0 + i\Omega}, \quad a_{\mathrm{gr}} = \frac{d\Omega}{d\psi} = \frac{q(1 - \varepsilon^0)}{p\varepsilon^0 + 2i\Omega}. \tag{13}$$

From the fourth equation of system (9) we obtain an algebraic expression to calculate the spectrum of permissible values of the wave number ψ or, with allowance for (12), the oscillation frequency:

$$i\left(H^0 - \frac{1}{pr}\right)\psi - \frac{2}{prn}(1 - \sqrt{1 + in\psi}) = 1 - \exp(-iH^0\psi), \tag{14}$$

where

$$n = \frac{4q(1 - \varepsilon^0)}{(p\varepsilon^0)^2}.$$

Finding the roots of Eq. (14), we determine ψ in (12) and calculate the permissible frequencies Ω of small oscillations in a uniform fluidized bed. The frequencies are complex in the general case and their imaginary parts will, in accordance with (11), determine the time behavior of the perturbations. Here, if the imaginary part of the frequency turns out to be negative, then the amplitude of the oscillation mode will increase until nonlinear effects begin to have a significant effect on the behavior of the system. The subsequent behavior of the system cannot be described within the framework of a linear approximation. Under certain conditions the system will evidently enter a limiting cycle (of oscillation) which may be manifest by the establishment of a pistonlike fluidization regime. Under other conditions, nonlinearity should lead to loss of stability of the wave fronts, which begin to break up and result in the formation of bubbles and packets. In this case, the group velocity of the waves c_{gr} may be identified with the surfacing velocity of the bubbles in the bed. In terms of order of magnitude (about 0.25 m/sec), this was confirmed by numerical calculation with (13) and by comparison with experimental measurements of bubble surfacing velocity taken from [9].

To check the above theoretical findings, we conducted an experiment to measure low-frequency pulsations of the pressure drop in a fluidized bed, the frequency of these pulsations in a linear formulation coinciding with the frequency of waves of particle velocity and porosity. Here, we first recorded the head characteristic of the gas blower in two regimes. The pressure-drop pulsations were recorded with a strain gage and a loop oscillograph. The oscillograms were analyzed numerically by the method described in [10]. The fluidized bed consisted of a glass column with an inside diameter of 58 mm and a perforated grate. The particles were glass beads 0.5 and 0.9 mm in diameter and corundum beads with an effective diameter of 0.5 and 0.63 mm. As the air-flow source we used two gas blowers with quite different head characteristics. The height of the loose bed was 100 mm in all of the tests. Statistical analysis of the oscillograms revealed the presence of low-frequency (< 10 Hz) oscillations of pressure drop in the bed. We then numerically found the real and imaginary parts of the complex frequencies from (14) and (12) and known parameters of the head characteristics K_1, K_2, and K for the specific conditions of the experiments. We considered only those frequencies with a negative imaginary part. The empirically obtained spectra of oscillation frequencies in the bed naturally turned out to be more saturated than the theoretical spectra, which is due to the presence of different mechanisms of oscillation generation in the actual bed besides the mechanism examined theoretically. Nevertheless, oscillation modes with frequencies near those calculated theoretically could be reliably picked out in the experimental spectrum. The results of comparison of the theory with the experiment are shown in Fig. 1, from which it is evident that the difference between the experimental and theoretical frequencies of self-sustained oscillations in different beds is not greater than 20%.

NOTATION

t, z, time and vertical coordinate, respectively; v, particle velocity; u, velocity of fluidizing gas; ε, porosity; g, acceleration due to gravity; F, force of hydrodynamic interaction between gas and particles per unit of bed height; h, height of fluidized bed; Δp_e, head created by source of flow of fluidizing gas; Δp_t, hydraulic resistance of supply main; d, particle diameter; ρ_0, μ_0, density and absolute viscosity of the fluidizing gas; ρ_1, density of the particle material; k_1, k_2, k_3, constants of the head characteristic of the gas blower; k_4, constant of the hydraulic resistance of the supply main; τ, η, dimensionless time and vertical coordinate; U, V, H, dimensionless gas velocity, particle velocity, and bed height, respectively; ϕ, b, B, K_1, K_2, and K, complexes determined in (5); ψ, dimensionless wave number; Ω, dimensionless frequency; a_p, a_{gr}, phase and group sonic velocities; p, q, r, complexes determined in (10). Indices: subscript 0, state of loose bed; superscript 0, state of homogeneous fluidization; ', deviation from state of homogeneous fluidization; $i = \sqrt{-1}$.

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CALCULATION OF MASS TRANSPORT IN NONISOTHERMAL EVAPORATION OF LIQUIDS FROM CAPILLARIES WITH CONSIDERATION OF VARIABLE VISCOSITY OF THE VAPOR–GAS MIXTURE

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The principles of vapor transport in a cylindrical capillary with temperature gradient are studied. Expressions are found for vapor flux and pressure of the mixture above the liquid meniscus in various evaporation regimes.

We will consider a cylindrical capillary of radius r, filled by a liquid, from the open surface of which evaporation occurs. We direct a coordinate axis from the mouth of the channel ($x = 0$) toward the liquid meniscus ($x = l$). We assume that the partial vapor pressure at the channel mouth P_{01} is constant and always less than the saturated vapor pressure at the liquid meniscus temperature $P_S[T(\ell)]$. The temperature varies along the capillary axis linearly, $T(x) = T_0 + \nabla Tx$. The binary gas mixture into which the liquid evaporates consists of molecules of vapor (first component) and gas (second component). We will perform the analysis with the assumption that the medium is continuous ($Kn \ll 1$) and that the flow of the vapor–gas mixture within the capillary is steady-state and one-dimensional. Thermodiffusion and barodiffusion components of the flow will not be considered because of their smallness.

In the general case in which no limitations are imposed on the vapor transport regime and it is necessary to consider both the hydrodynamic flow of the vapor–gas mixture and interdiffusion of the components, the densities of the steady-state vapor and gas flows in a coordinate system fixed to the capillary are described by the following equations:

$$j_1 = -D(x)\frac{d\rho_1(x)}{dx} - \frac{r^2\rho_1(x)}{8\eta(x)}\frac{dP(x)}{dx} = \text{const}, \tag{1}$$

$$j_2 = -D(x)\frac{d\rho_2(x)}{dx} - \frac{r\rho_2(x)}{8\eta(x)}\frac{dP(x)}{dx} = 0. \tag{2}$$

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